A non-contacting vertical suspension for the vehicle shown above is being considered. The vehicle has controllable electro-magnets as shown that create a magnetic force between the vehicle and a ferromagnetic material on the guideway. The vehicle magnets are to be current controlled and an approximate expression for the electromagnetic force is: \( F_M = \frac{\alpha I^2}{(\Delta y)^2} \) where \( I \) is the control current and \( \Delta y \) is the “gap” (\( \Delta y = y_g - y_v \)). \( y_v \) is the vehicle vertical position and \( y_g \) is the vertical position of the guideway. Irregularities in the guideways construction can lead to a changing vertical position of the guideway as the vehicle passes over it. We will consider the guideway’s vertical position as a disturbance, \( y_g(t) \).

A simple 1 DOF vertical model for this system is: \( M\ddot{y}_v = \frac{\alpha I^2}{(y_g - y_v)^3} - Mg + F_d \) where \( F_d \) is an external disturbance, e.g. aerodynamic loads.

(a) Find the steady-state current, \( I_o \), required to hold the vehicle statically at a gap of \( \Delta y = h_o \) (assume \( F_d = 0 \)).

(b) Linearize the system about the nominal operating condition of \( I = I_o, y_g = 0, F_d = 0, y_v = h_o \), i.e \( I = I_o + \delta I \)

derive a linear differential equation relating \( \delta I, \delta y_v, \delta y_g, \) and \( \delta F_d \) where:

\[
\begin{align*}
\delta y_v &= \delta y_v + \Delta y_v \\
\delta y_g &= \delta y_g \\
F_d &= \delta F_d
\end{align*}
\]

(c) Discuss the stability of the open-loop system.

(d) A closed-loop proportional controller: \( \delta I = K_1 \Delta y + K_2 (\delta y_g - \delta y_v) \) has been proposed. Discuss the stability of this system.

(e) A PD controller of the form: \( \delta I = K_1 \Delta y + K_2 \Delta y \) is being considered:

i. Is the closed loop stable?

ii. What is the steady-state vehicle position, \( y_{ss} \), if \( y_g = 0 \) and \( F_d \) is a step disturbance of magnitude \( F_{d,s} \).
(f) A PID controller of the form: $\delta I(s) = K \left[ 1 + T_d s + \frac{1}{T_i s} \right] \Delta y(s)$ is being considered:

i. Find the transfer functions $\frac{\delta y_v(s)}{y_g(s)}$ and $\frac{\delta y_v(s)}{F_d(s)}$

ii. Sketch the root locus as a function of $K$

iii. What is the final value of $y_v$ if $y_g = 0$ and $F_d = F_{d_u}$

(g) A gap sensor, $\Delta y = y_g - y_v$ is available. How are we going to get $\Delta \dot{y}$?