A position servo for a rotational inertia is to be designed using classical servo theory. The resulting design is then to be implemented digitally. The figure below is a schematic of the digitally controlled system.

Figure 1

(a) "continuous design"

(i) find the transfer function:

\[ \frac{\Theta(s)}{T(s)} = \frac{A G_p(s)}{T(s)} \]

(ii) The "actuator" is modelled as a constant gain block, i.e.,

\[ T(s) = K_T u(s) \]
we want to design a compensator, \( G_c(s) \), such that the system shown below is well behaved:

![Block diagram](image)

**Figure 2**

Let \( K_1 = 1.0 \), and \( I = 1.0 \), evaluate the stability of the closed-loop system if a proportional controller is used, i.e.,

\[
G_c(s) = K
\]

(iii) Evaluate the stability of the closed-loop system if a lead/lag controller is used, i.e.,

\[
G_c(s) = \frac{K(s+1)}{(s+3)}
\]

(b) "Digital implementation"

The lead/lag controller analyzed in (iii) is to be implemented digitally. The digital controller is composed of a discrete version of \( G_c(s) \) followed by a D/A converter (zero order hold). It was observed that for a large value of the "sampling interval" \( \frac{1}{T} \), that the closed-loop system shown in figure 1 was unstable.
An approximate representation of the effect of the zero order hold in Figure 1 is shown below:

\[ \frac{K(s+1)}{(s+3)} \rightarrow \frac{2/T_s}{s + \frac{2}{T_s}} \rightarrow \frac{1}{s^2} \rightarrow e \]

Figure 3

where \( T = \) sampling interval

(iv) analyze the effect of small sampling intervals, let \( T_s = .2 \) sec., and discuss the stability of the closed-loop system shown in Figure 3 for \( 0 < K < \infty \)

(v) analyze the effect of large sampling intervals, let \( T_s = 200 \) sec, on the closed-loop stability

(vi) fix the gain (K) of the lead/lag compensator analyzed in (iii) and analyze the stability of the system shown in Figure 3 as \( T_s \) is varied from 0 to \( \infty \).