A human operator applies torque $\tau_1$ to a master control arm having inertia $I_1$ and angular position $\theta_1$, which in turn controls a slave robot arm having inertia $I_2$ and angular position $\theta_2$. Electrical input $i$ to the slave arm's motor is $K(\theta_1 - \theta_2)$, as shown above. The motor characteristic may be approximated by sets of torque-speed straight-lines, evenly spaced as a function of input $i$, as shown below. Assume $\frac{d\tau_2}{di} = 1$.

1. What is the transfer function ($\theta_2/\tau_1$)?

2. Assume the characteristics of the human muscle, $\tau_1(\theta_1, N)$, are similar to those of the electric motor, $\tau_2(\theta_2, i)$, where $N$ is the intensity of nerve impulses from the brain to the muscle. Assume $C_1 = C_2$ and $\frac{d\tau_1}{dN} = 1$. Then what is the transfer function ($\frac{\theta_2}{N}$)?

3. Investigate (through root locus or otherwise) the stability if the human generates nerve impulses $N$ in proportion to observed slave position error, that is $N = K_h(\theta_2^* - \theta_2)$, where $\theta_2^*$ is a desired position.

4. How would this differ if the human could willfully control master position $\theta_1$ as a lagged function of slave position error, that is $\theta_1 = K_h(\theta_2^* - \theta_2)/(Ts + 1)$?

5. What if the human were also sensitive to rate of change of $(\theta_2^* - \theta_2)$?

6. From your intuition which is the most likely, (3), (4) or (5)? Why?

7. If $I_1$ and $I_2$ were small and the servo gain $K$ large relative to $T$, sketch the likely Bode (gain-phase) plot for $\theta_2/(\theta_2^* - \theta_2)$ in case (5).