January 1995

Mechanical Engineering Dept.
Oral Qualifying Exam
System Dynamics and Controls
1/23/95

Instructions:

This problem is based upon the temperature control mechanism you have been given. What we are most interested in is your ability to connect your knowledge of systems and control with this physical example.

You have 20 minutes to study the following document, and the temperature control valve itself. Large parts of this document are tutorial, in that they explain the operation of the valve.

The best use of your 20 minute preparation time is to be sure that you solidly understand how the valve works. Do not race to answer the example questions at the back of this problem statement. Simply use these questions as a check of your understanding. We may or may not use these specific questions in the actual exam. In preparing, emphasize physical understanding over detailed computation.

Be sure to show up promptly at the scheduled time for your oral exam in the room indicated on the posted schedules.

At the end of the exam, please return to us all materials, including this problem statement, and any notes you make.

Stay as relaxed as possible.
1 Introduction

This problem concerns a temperature regulating valve. You are provided with a cutaway version of this valve in both the preparation room and in the exam room itself. The valve is also shown in cross-section in Figure 1, with major subassemblies identified. These are discussed in detail below, following an overview of the intended use of the valve.

![Temperature regulating valve diagram]

Figure 1: Temperature regulating valve in cross section. Major sub-assemblies are identified as shown.

You will notice that the valve has three connections (or ports) for soldering to 1/2" copper pipe, as commonly used in residential plumbing. A section of such pipe has been included in your materials for reference. These ports are labeled cold, hot, and mix. The port labeled cold is connected to the residential source of cold water, which is at a temperature of 20 °C. The port labeled hot is connected to water exiting a boiler heat exchanger. This water is at a temperature of 90 °C. These two sources of water may be assumed to be of equal pressure. The function of the valve is to mix the hot and cold water in the proper amounts so that the water leaving the mix port is at a desired intermediate temperature, usually in the range of 50 to 70 °C. The rotary knob on top of the valve is used to adjust the temperature of the mixed water leaving the valve, within the above range.
2 Detailed description

Refering again to Figure 1, we describe the valve parts as follows. Starting at the top is the temperature adjustment knob. This knob turns a threaded rod (setpoint rod) which pushes on the back of the sensor/actuator. The actuator plunger pushes on the spring-loaded cold-water poppet valve. You have been provided with a wood dowel which can be used to push open the cold-inlet poppet valve. Since the spring which closes this valve is relatively stiff, we recommend that you use the dowel rather than your finger for this task.

The heart of this assembly is the sensor/actuator. This is shown in more detail in Figure 2. Two views are given. The first view is at low temperature (less than 50 °C), where the actuator plunger is fully retracted. The second view is at high temperature (90°C), where the actuator plunger extends by about 3 mm. This motion is driven by a thermally-sensitive wax contained within the sensor body which expands significantly with temperature. Because the motion is driven by an essentially incompressible fluid, the extension of the plunger can be considered to be independent of any typical loads imposed on it. That is, the actuator/sensor can be thought of as a position source which depends only upon its internal temperature. This assembly is thus simultaneously the system sensor and actuator. A similar type of thermostat is used to control the temperature of automotive engine cooling systems.

3 Simplified linear modeling

Throughout this problem, we will consider the valve from two perspectives. The first is the real system, which if considered in detail, is highly nonlinear, due to parts which can come in and out of contact, fluid flow through orifices, fluid mixing processes, saturation of the sensor/actuator, and numerous other real-world effects. The second perspective is a simplified linear analysis which captures the basic valve operating mechanisms. This linear model is introduced in more detail below. In answering questions in this oral, please make clear which model you are considering.
In order to capture the basic operating principles of the valve in a simple linear setting, we consider the valve behavior about a steady-state regulation operating point. That is, suppose the valve is operating at the flow required for a person taking a shower, with the adjustment knob set so that the mix temperature leaving the valve is about 60 °C. In this case, the poppet valve is partially open, and water is being forced in through both the hot and cold ports in order to mix to the desired temperature. We then consider small perturbations about this operating point, e.g., the temperature of the hot water rises by 1 °C, or the adjustment knob is turned by 0.1 radian. Such small changes about an operating point are referred to as incremental. In this case, a linear model is adequate to describe the system operation.

Let us define \( x_s \) as the incremental change in setpoint rod position. We further define \( x_a \) as the incremental change in actuator plunger extension. Both of the above increments act to further open the poppet valve, which is defined as having an incremental opening \( x_p \). That is,

\[
x_p = x_s + x_a.
\]  

(1)

All of the above incremental positions are defined in millimeters.

We define the incremental change in mix temperature as \( T_m \), and the incremental change in hot water inlet temperature as \( T_h \), in units of °C. The cold water temperature is assumed fixed, and thus its incremental value is zero. Further information about the valve operation is that the mix temperature \( T_m \) is affected by \( T_h \) and \( x_p \) as

\[
T_m = T_h - g_p x_p,
\]  

(2)

where \( g_p \) is the poppet position-to-temperature gain coefficient in units of degrees °C per millimeter.

Let us define the actuator incremental internal temperature as \( T_a \). Due to the thermal mass of the actuator, this temperature does not instantaneously equal the mix temperature \( T_m \). We will model the process of equilibration between these two temperatures by the first-order differential equation

\[
\frac{dT_a}{dt} = c(T_m - T_a),
\]  

(3)

where \( c \) is a thermal equilibration coefficient with units of sec\(^{-1}\). We further assume that \( T_a \) does not in turn affect the mix temperature.

Finally, the actuator incremental extension \( x_a \) is related to \( T_a \) by

\[
x_a = g_a T_a,
\]  

(4)

where \( g_a \) is the sensor-extension gain coefficient in units of millimeter per degree °C.

Given the above information, we assert that the linear model can be represented by the block diagram shown in Figure 3.
4 Linear model typical questions

With respect to the linear model above, be prepared to answer questions like the following:

- Explain how to construct the block diagram of Figure 3, given the information presented above.

- What experiments could be done to determine the numerical values of the parameters $c$, $g_a$, and $g_p$?

In the following questions, assume that the system parameters take the values $c = 0.25 \text{ sec}^{-1}$, $g_a = 0.06 \text{ mm/°C}$, and $g_p = 50 \text{ °C/mm}$. Be prepared to answer questions such as the following.

- Calculate and graph the response in $T_m$ to a 1 °C step in $T_h$. Calculate and graph the response in $x_a$ to a 1 °C step in $T_h$.

- Calculate the transfer function $T_m(s)/T_h(s)$. Plot the poles and zeros of this transfer function. Sketch a Bode plot representing the frequency response.

5 General questions

Be prepared to answer general questions such as the following.

- Explain physically how the valve works.

- Explain physically (not in equations), the sequence of events which occur when the adjustment knob is turned in by a quarter turn.

- Why is this a feedback system? How is it superior to a fixed-adjustment mixing valve?

- Why are there fins on the sensor/actuator? What would happen to the valve response characteristics if these fins were filled in with epoxy?
Figure 3: Linear model block diagram.