May 1999

Systems Oral Exam

May 26th, 1999

Note: The exam is twenty minutes long. You will have twenty minutes beforehand to read the exam and devise answers to the questions.

A rotor with moment of inertia \( J = 1 \text{ kg-m}^2 \) and angular velocity \( \omega \) radians per second is subject to an externally applied torque \( T_{\text{ext}} \) and to an internal frictional torque \( T_f(\omega) \). The frictional torque has the following form (Coulomb friction): it always opposes the direction of motion, taking the value \( T_f(\omega) = -1 \text{ N-m} \) when \( \omega > 0 \) and \( T_f(\omega) = +1 \text{ N-m} \) when \( \omega < 0 \). \( T_f(\omega) = 0 \) when \( \omega = 0 \).

1. Write down the equation of motion for the rotor. Is this equation linear or nonlinear?

2. Suppose that the rotor always operates in a regime where \( \omega > 0 \). Can it be described by a linear equation of motion in this region? Write down a transfer function for the rotor assuming that the rotor always has \( \omega > 0 \).

3. Discuss the advantages and drawbacks of open-loop control for the rotor.

4. Devise a closed-loop controller to control the rotor to \( \omega_d = 100 \) radians per second. Assuming the controller operates in a regime where \( \omega > 0 \), write down its transfer function and sketch a root-locus diagram.

5. Now suppose that the controller operates in a regime where \( \omega < 0 \) and \( \omega_d = -100 \) radians per second. How do the transfer function and root-locus diagram in 4 change?