This problem considers the piezoelectric-actuated system shown in Figure 1. Here a massless link pivots without friction as driven by the piezoelectric actuator at a distance \( l_1 \) from the pivot. A mass \( M \) and spring \( k_1 \) are attached at the right-hand end of the link. The spring \( k_1 \) is pre-stretched with sufficient tension to guarantee that the hemisphere at the end of the piezo remains in contact with the link at all times. The motions \( x_1, x_2 \) are about the equilibrium established by this preload. All motions are small. Ignore gravity.

We further assume that the piezo can be modeled as shown in Figure 2. Here, motion can only occur in the \( x_1 \) direction. The force source \( F_p \) depends upon the applied piezo voltage \( E_p \) as \( F_p = GE_p \) where \( G \) has units of \( N/V \). The spring \( k_p \) represents the finite piezo stiffness.

a) Calculate the transfer function \( X_2(s)/E_p(s) \). Write an expression for the natural frequency \( \omega_n \). Sketch a Bode plot for this transfer function. Note that your expression and plot will include the variables \( G, k_p, M, k_1, l_1, l_2 \). Note that there is no damping in the problem.

b) Use power considerations to derive an expression for the piezo current \( i_p \) in terms of \( x_1 \).

c) Design a controller of your choice to control position \( x_2 \) in order to track a reference \( x_{2R} \). This controller must have a bandwidth on the order of 10 \( \omega_n \), where \( \omega_n \) is the natural frequency calculated in a). The control input is \( E_p \).

d) **Optional. Only look at this if you have completely understood a) - c) above!**

Now assume that the second-order mode of \( X_2(s)/E_p(s) \) has a damping ratio \( \zeta = 0.1 \). Further assume that we apply a controller of the form \( K/s \). What is the maximum allowable value of \( K \) for stability? Sketch a Nyquist plot which allows you to demonstrate this result.
Figure 1: System configuration

\[ F_p = G E_p \]