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Massachusetts Institute of Technology
DEPARTMENT OF MECHANICAL ENGINEERING

SYSTEM DYNAMICS AND CONTROL
WRITTEN EXAM, JANUARY 1989

Things to remember:

- You have 1 hour.
- This is an OPEN BOOK test.
- There is just one question (but with many parts).
- This exam assumes you have a calculator. If you don’t, do not panic, just show how
  you get the numbers and put down approximate estimates.
- Round numbers off to two or three significant digits.
- Relax!

Introduction

You are in Harry’s HotRod dealership when you come upon your dream car. Suddenly the
urge hits to do a little modelling of the car’s suspension system so you pull out your pencil and
get to work. Because you only want to get some ballpark information, you develop a “quarter-
body” model of the car which assumes that the car body sits as a lumped mass on a single
suspension consisting of one spring and one shock absorber in parallel, resting on a single axle
and tire. Assume that the axle and tire can be modelled as a spring-mass combination
with no damping (see Figure).

![Diagram of a quarter-body model]

Figure: Your dream car as a quarter-body model.

You ask Hank, the salesman, for some of the pertinent specs on the car’s suspension
system. He says “Just a minute, I’ll have to check with the manager” and runs off. Half an hour
later (Don’t you love car dealers?) he returns with the following table.
Hank explains that the tire stiffness is a little more complicated and shows you the following formula.

\[ F = q x^2 \]

where \( F \) is force on the tire, \( x \) is the tire deflection, and \( q \) = a constant = \( 1.56\times10^6 \) N/m².

**Task 1**
Rolling, pitching, higher dynamics

Justify the assumptions made in going from the actual car to the quarter-body model shown in the figure. What do you lose? What do you gain?

\[ \text{low } q, \ x \]

**Task 2**
You are interested in linear models only. Develop a linear spring model for the tire about a sensible operating point. What is the resulting spring constant, \( K \)? (Numerical values please.)

**Task 3**
Using the modelling method of your choice, develop a set of state equations for the car suspension system. Your answer should be in the form

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]

where \( x \) is the state vector and \( y \) is the output vector. Assume that you are interested in two outputs, the velocity of the car body and the acceleration of the car body (which are related to driver comfort). The choice of a state vector is up to you.

Be sure you are clear in writing down the steps of your modelling procedure and equation derivation.

**Task 4**
Write a new output equation where the single output, \( y \), is the car body position.