May 1989

DOCTORAL WRITTEN QUALIFIERS

CLOSED-BOOK

We consider the problem of controlling the vertical attitude of a rocket (for simplicity, we consider only a planar version). A block diagram representation of the closed-loop system is

\[ \Theta_d(s) = 0 \]

The dynamics of the actuator (which varies the angle of a thruster at the basis of the rocket) can be written as

\[ G_a(s) = \frac{1}{(s + \omega_n)^2} \]  

(1)

The angle \( \Theta \) and the angular velocity \( \dot{\Theta} \) are available for measurement. You decide to design a P.D. controller of the form

\[ G_c(s) = K(s + 2) \quad \text{where } K > 0 \]

(a) With \( \omega_n = 10 \), draw the corresponding root locus. Find the upward gain margin and the downward gain margin for \( K = 600 \).

Determine approximately the phase margin of your design. Interpret this information physically.
(b) Consider instead the use of a slow actuator (e.g. fixed thrusters along independent directions but time-varying thrust), still of the form (1) but with a different $\omega_n$. Show that if the actuator dynamics is too slow, the closed-loop system will not have any stable region. Find the range of $\omega_n$ which will make the closed-loop system always unstable.

You realize that due to the many sources of vibrations in the rocket, your measurement of $\hat{\theta}$ is too noisy. You decide instead to use the following controller.

$$G_c(s) = K \left( \frac{\alpha s}{s + \alpha} + \lambda \right)$$

where $\alpha = 100$

(c) Explain why the above controller can actually be implemented using measurement of $\theta$ alone.

(d) Wind tunnel tests indicate that at take-off, disturbances $d$ are concentrated in a frequency range of $90 < \omega_d < 110$ (rad/sec).

How could this information be used to adjust the value of $\alpha$?

(e) Can you suggest a more sophisticated expression for $G_c(s)$?