A robot arm joint, together with a linearized model that ignores gravity effects, is shown below:

The arm is driven by an electric motor (with armature resistance $R_m$ and inductance $L_m$), which produces a torque proportional to armature current; $T = K_T I_a$. The motor is connected to the arm through a gear train (ratio $n$) as shown, and the arm is modeled as a fixed inertia $J$ in bearings with a rotational damping factor $b$. (Note that $J$ and $b$ represent the values "reflected" through the gear train to the motor shaft.) The motor back-emf $E_m$ is proportional to the shaft angular velocity $\dot{\theta}_m$.

(a) If the motor is an energy conserving transducer, find the relationship between $K_m$ and $K_T$.

(b) From the block diagram derive a set of state equations, and a single output equation describing the arm dynamics. (Do not attempt to model from first principles.)

(c) Find an expression for the transfer function relating the arm angle $\theta_L$ to the applied voltage at the motor terminals $E_a$. 
The robot arm angular position is controlled by a linear control system that monitors the angular position \( \theta_L \) as shown below:

For simplicity assume the following values (with appropriate units): \( J = 1, \ b = 1, \ L_m = 1, \ R_m = 4, \ K_T = \sqrt{2}, \ n = 1/20. \)

(d) Show that the plant transfer function reduces to

\[
\frac{\theta_L(s)}{E_a(s)} = \frac{0.0707}{s(s + 2)(s + 3)}
\]

(e) Sketch magnitude and phase Bode plots for the plant. You may use straight line approximations but label all break (corner) frequencies and line slopes. Determine the system response to an input

\[ E_a(t) = 3\sin(2t) \]

(You may be approximate – do not solve the frequency response function – use your plots.)

(f) Assume that the compensator is a proportional gain \( G_c(s) = K \).

(i) Find the closed loop transfer function for the system.

(ii) Sketch a root locus for the system, and comment on the the closed loop stability as the gain \( K \) is increased.

(g) If the closed loop system is given a unit step input, \( \theta_c(t) = 1 \) for \( t > 0 \), find the steady-state error in the arm response as a function of \( K \).

(h) Sketch a Nyquist diagram for the system under proportional control. Show how you would estimate the system gain and phase margins from your plot.

(i) Now assume that the compensator is a P-D controller of the form \( G_c(s) = K(1 + 0.2s) \). Sketch a root locus plot for this system. Comment on the system stability as a function of \( K \).

(j) Is there any point to including integral action (that is using a PID controller) for this plant?