May 1994

Massachusetts Institute of Technology
Department of Mechanical Engineering
Doctoral Qualifying Written Examination
System Dynamics and Control
May 1994

- You have 1 hour.
- This is a CLOSED BOOK test.
- Keep things simple and avoid doing unnecessary work.
- As much credit will be awarded for presenting an insightful, well-reasoned approach as for arriving at "correct" answers to the questions.

A gantry-type robot is being developed at the Technology Center of GMFC Inc. As shown in Figure 1, the robot system consists of a large frame, called "gantry", and an arm link that traverses along the x axis. The development team had technical difficulties in designing the control system of the long traverse axis. They have modified the design and built prototypes several times, but desired performance has not yet been obtained. Your job is to analyze the system by building mathematical models that elucidate the problems they faced.

![Diagram of Gantry-Type Robot]

Figure 1
1. The first design is depicted in Figure 1. The arm link slides on the linear guide, being driven by a DC motor through a steel belt. To control the position of the arm link in the x direction, a PD control loop was formed by taking a feedback position signal from a shaft encoder mounted on the motor axis. The system was stable for a broad range of PD gains, but the positioning accuracy at the arm tip was poor due to the compliance of the long steel belt as well as friction and disturbance at the transmission mechanism. To improve accuracy, the development team installed a high precision linear encoder along the linear guide in order to measure the exact position of the arm link in the x direction. A PD control loop was formed by using the linear encoder as the feedback signal. It turned out that the PD control system became unstable as the proportional gain increased. You were asked why the PD control from the motor shaft encoder was stable and the one from the linear encoder was unstable. Use the following notation shown in Figure 2 to analyze the system:

\[
\begin{align*}
x_1 &= \text{actuator displacement} \\
x_2 &= \text{arm link displacement} \\
K &= \text{stiffness of the steel belt} \\
m_1 &= \text{effective inertia of the motor rotor and gearing reflected to the steel belt} \\
m_2 &= \text{inertia of the arm link} \\
b_2 &= \text{viscous damping at the linear guide (acting between the arm link and the gantry)} \\
F &= \text{actuator force generated by the DC motor}
\end{align*}
\]

The actuator dynamics is governed by:

\[
F = u - K_b x_1
\]

where \( u \) is control input, and \( K_b \) an effective back emf constant. Assume small \( b_2 \).

![Figure 2](image)

(1-1) Obtain the transfer function from input \( u \) to actuator displacement \( x_1 \) as well as the one from input \( u \) to arm link displacement \( x_2 \).

(1-2) Sketch root loci for the two PD control systems described above and explain why the feedback from the motor shaft encoder was stable and the one from the linear encoder was unstable. Plot open-loop poles and zeros at appropriate positions which make sense physically.
2. Since the compliance of the steel belt was the major source of poor performance, the development team decided to replace the steel belt by a heavy-duty lead screw, which is an order-of-magnitude stiffer than the steel belt. As a matter of fact, the encoder reading at the actuator side agreed with the linear encoder measuring the arm link position even under a large load. This implies that the compliance of the lead screw is negligible. A PD control loop using the linear encoder signal was formed for this modified design. The stability of the system was achieved even for high gains. However, unwanted vibrations were observed at the arm link due to the vibration of the gantry structure (See Figure 3). For increased feedback gains, the compliance of the gantry was no longer negligible. The development team measured the horizontal stiffness of the gantry by applying a force $f$ to the top of the gantry, as shown in Figure 3. The identified stiffness of the gantry, $K_g$, was significantly lower than that of the lead screw, the arm link, and other mechanical parts.

To analyze the system, consider the simplified model shown in Figure 4 and use the following notation:

- $b_1 =$ viscous damping of the gantry
- $b_2 =$ viscous damping of the linear guide
- $x_0 =$ horizontal deflection of the gantry,
- $x_2 =$ arm link position viewed from an inertial reference frame,
- $M_0 =$ effective inertia of the gantry,
- $m =$ inertial load of the actuator including the arm link, the motor rotor and the lead screw

Note that in the simplified model, the actuator force $F$ acts not only on $m$, but also on $M_0$ as a reaction force. The back emf constant of the actuator is the same as $K_b$ in Problem 1 above. Assume small $b_1$ and $b_2$. 
(2-1) Obtain the transfer function from input $u$ to the output of the linear encoder, and show why the PD control system was stable. Note that the output of the linear encoder does not necessarily mean the actual arm link position viewed from an inertial reference frame.

(2-2) Explain why the arm link motion was vibratory for a high proportional gain, although the linear encoder output did not indicate such large vibrations. (Hint: Consider the transfer function that relates $x_2$ to the linear encoder output, and examine how the transfer function changes the zeros associated with the response of $x_2$.)