January 1997

Massachusetts Institute of Technology
Department of Mechanical Engineering

Written Qualifying Examination
System Dynamics and Controls

January 1997

- You have one hour to complete this exam,
- This exam is closed book and closed notes,
- Relax.

1 Problem: A Magnetically levitated rotor

Figure 1-a shows a rigid rotor levitated by two magnetic bearings producing the individual forces $f_1$ and $f_2$. The displacements associated with the points of application of these forces are denoted by $x_1$ and $x_2$. These points are located at distances $a_1$ and $a_2$ from the center of mass of the rotor. We will be concerned only with the dynamics of the rotor in the horizontal plane. (Do not be concerned with the lateral or any other motions of the rotor except the one mentioned). Figure 1-b shows the rotor when it is perfectly aligned with the axis of rotation.

First we will focus on one bearing. Let $f$ represent the force generated by this magnetic bearing. When a current $i$ flows into the coils of such a bearing, the force $f$ is not generated instantaneously due to the bearing dynamics. Let us assume that the linearized differential equations which relate the control current $i$, the displacement $x$, the magnetic flux $\phi$, and the momentum $p$ of the net inertia $m$ "seen" by this bearing are given by,

\begin{align}
\dot{\phi} &= -\kappa_1 \phi + \gamma x + bi \\
\dot{x} &= \frac{p}{m} \\
\dot{p} &= \kappa_2 \phi
\end{align}

Note that the displacement $x$ represents the deviation of the rotor from the axis of rotation, and the magnetic force $f$ is the time rate of change of the momentum,

\begin{equation}
f = \dot{p}
\end{equation}

The parameters $\kappa_1, \kappa_2, \gamma$ and $b$ are scalar constants.
Figure 1: A rotor levitated by two magnetic bearing systems

1.a State the key assumptions that led to the equations given above.

1.b What is the likely origin of the first order dynamics of \( \phi \)?

1.c Discuss the stability of this bearing. Justify your answer.

1.d Draw the corresponding pole-zero plot. Explain briefly.

1.e Determine the transfer function \( \frac{X(s)}{F(s)} \). \( X(s) \) and \( F(s) \) are the Laplace transforms of \( x(t) \) and \( f(t) \) respectively.

2.a Consider now the system of Figure 1 but neglect the dynamics associated with the generation of the forces \( f_1 \) and \( f_2 \). Derive the state equations of this system.

2.b Describe the energy storage in the system of Figure 1.

2.c Draw a pole-zero plot. Explain briefly.

3.a Consider again the system of Figure 1 but do include the dynamics associated with the generation of the forces \( f_1 \) and \( f_2 \). Draw a pole-plot for this system.

3.b Suggest a controller for the system of part (3) and comment briefly.