May 1997

Massachusetts Institute of Technology
Department of Mechanical Engineering

Written Qualifying Examination
Systems Dynamics and Control
May 1997

Things to remember

- You have one hour.
- This exam is closed book. It is two pages long.
- Keep your answers simple.
- Relax.

To compensate for sudden shifts of luggage during flight, an airline proposes to install a pitch-attitude autopilot in their planes. The block diagram for the autopilot is shown in figure 1. Luggage shifts will be modeled as a step disturbance $M_p(s) = M_0/s$, with $M_0$ no greater than 0.6.

1) Assuming that the system is stable, what value of $K$ is required to keep the steady-state error in $\theta$ less than 0.02 radians?

2) Draw a root locus with respect to $K$.

3) Based on your root locus, at what value of $K$ does the system become unstable?

4) Suppose that an acceptable steady-state behavior requires $K = 600$. Show that the system is unstable at this value of $K$ with roots at $-3, -14$, and $+1 \pm 6.5j$.

5) To make the system stable, you propose installing a rate gyro that measures $\dot{\theta}$, giving an output $K_T\dot{\theta}$. Take $K = 600$ as in (4). In figure 1, show how you would incorporate the rate gyro in the autopilot to stabilize the plane. Include any transfer functions in boxes.

6) Draw a root locus with respect to $K_T$ for the new block diagram you constructed in (5).
7) What is the maximum damping factor for the complex roots that can be obtained with the configuration of (5)?

8) What is the value of $K_T$ that gives the maximum damping factor in (7)?

9) Draw amplitude and phase Bode plots for the autopilot, and indicate the gain and phase margins on the plots.

10) Suppose that the attitude sensor suffers from a time delay $\tau$. In terms of the Bode plots in (9), discuss the stability of the system as $\tau$ increases.

Figure 1