A Steering Autopilot for Recreational Boats

More and more recreational boats, sail and power, are using autopilots to maintain a constant course (compass heading) in the presence of disturbances from wind, waves, and currents. In this question we examine the operation of a simple add-on autopilot that attaches directly to the steering wheel of a boat. Such units can now be purchased for $400 and up. Commercial units are usually digital in nature, but we will assume continuous control action in this question. The figure below is taken from a brochure for the Raymarine ST4000+ MkII sailboat autopilot.

The unit consists of three main components: (1) a “control head” which contains the user panel, controller and motor-drive electronics, (2) an integrated motor/wheel-drive unit that attaches to the ship’s (steering) wheel and uses a toothed-belt to turn the wheel in response to commands from the control unit, and (3) an electronic “flux-gate” compass that provides a voltage proportional to the vessel’s magnetic course. The compass acts as the sensor that indicates the true course.
The autopilot is a feedback control system that detects an error in the desired course and takes action through the boat’s rudder to correct the course.

The following is a verbal description of simplified dynamics of ship steering and autopilot operation. In modeling the system you may find that some parameters are given, and some are not. In addition some parameters are given numerical values, others are not. This is deliberate - you should “fill-in-the-gaps” as necessary. Assume that the boat is moving forward at a constant speed.

(i) When the rudder is moved by an angle $\Delta$, the fluid flow across it generates a torque that rotates the vessel about an on-board axis according to a (linearized) differential equation:

$$J \frac{d\Omega}{dt} + B\Omega = K_r \Delta$$

where $\Omega$ is the angular velocity of the boat about its axis of rotation, $J$ is the moment of inertia about the axis of rotation, and $B$ is a viscous rotational friction coefficient, and $K_r$ is a constant. Assume $J/B = 2$ sec.

(ii) The torque $T$ required by the helmsman (or autopilot) at the steering wheel, to move the rudder to an angle $\Delta$ may be assumed to be directly proportional to $\Delta$.

Some of the characteristics of the autopilot are:

(iii) The control head compares the desired heading ($\theta_d$), and actual ($\theta$) heading (from the compass) and produces a voltage proportional to the error. The error is processed by a linear controller, with transfer function $G_c(s)$ to produce a command voltage $v_c$.

(iv) The command $v_c$ is processed by a linear power amplifier (that has its own internal dynamics), and produces a motor current $I_m$. Sinusoidal frequency-response measurements on the power amplifier show that has a Bode magnitude plot (relating output current $I_m$ to input voltage $v_c$) typical of first-order lag, with a corner (-3dB) frequency of 20 Hz. and a high frequency asymptotic slope of -20 dB/decade.
(v) The autopilot motor is a permanent-magnet dc motor. The motor drives the steering wheel through a toothed belt with a step-down ratio of $N$.

(vi) The flux-gate compass electronic circuitry includes a simple first-order low-pass filter to reject noise. A simple test made by suddenly rotating the compass by 90° showed that it took approximately 4 seconds to reach its new steady-state value, and changed the output by 4.5 volts. The compass output is 0 volts when oriented to magnetic north.

(a) Use the information given above (together with any additional factors you may need) to draw a block diagram of the closed-loop system steering system, with the desired ship's heading $\theta_d$ as the input, and the actual heading $\theta$ as the output. Each block in your diagram should be an $s$-domain transfer function. You may assume that the controller has an undetermined transfer function $G_c(s)$.

(b) In practice the closed loop system will be subjected to external influences such as wind gusts, waves and currents. Modify your block diagram to show how you would include a disturbance input, representing the combined effects of wind and waves on the boat's heading. Make clear what the units associated with the disturbances are.

(c) Derive closed-loop transfer functions (i) relating the course $\theta$ to the desired course $\theta_d$, and (ii) relating the course $\theta$ to the disturbance input.

(d) Assume a proportional controller for $G_c$, determine the numerical values of all the system poles and zeros, and sketch a pole-zero plot for the system.

(e) Discuss the closed-loop stability of the system using root-locus arguments. Will it go unstable?

(f) The Raymarine ST4000+ installation manual describes a calibration procedure to set the "rudder gain" (controller overall gain). It requires you to sail, making 40° step course adjustments, and adjusting the gain to achieve about 2° of overshoot in coming to the new course.

Make an $s$-plane sketch of the approximate location of the dominant closed-loop poles.
in this condition.

(g) Do you see any advantage of modifying the controller \( G_c(s) \) to a P-I (proportional + integral) form?

(h) Most manufacturers offer an optional rudder-angle (\( \Delta \)) sensor to provide an additional input to the controller. The installation manual states that use of this sensor "will optimize the response". Discuss how you would use this additional information to enhance the autopilot performance. Draw a block diagram showing how you would incorporate the additional input into the control scheme.