Problem 1  (40%)

Consider the following fourth-order system with a unity feedback.

(1-a). Plot the open loop poles on a complex plane, and draw a root locus. (You can draw the exact root locus for this system without use of a computer.)

(1-b). Using the root locus plot, graphically obtain the feedback gain $K$ that makes the closed loop system marginally stable.

(1-c). Verify the result of Part (1-b) by using the Routh-Hurwitz stability criterion.

(1-d). Sketch the step response of the marginally stable closed loop system. What is the peak time? Indicate the peak time in your step response.
Problem 2 (60 %)

Shown below is a one degree-of-freedom robot arm with a deburring end effector. When deburring a workpiece, a reaction force $F_d$ acts on the deburring tool at the endpoint of the robot arm. This reaction force acts as a disturbance to the robot position control system regulating the robot endpoint position, $y$. The length of the arm is $L$, and its inertia about the joint axis is $J$. An actuator generates torque $\tau$ on the joint axis. The bearings supporting the joint axis have a viscous damping of $b$. Assume that the arm is a rigid body and that it is almost at the upright position; the joint angle $\theta$ varies within a small range near 90 degrees. Answer the following questions.

![Diagram of one degree-of-freedom deburring robot]

Figure 2 Schematic of one degree-of-freedom deburring robot

(2-a). Obtain the equation of motion and show that, in Laplace form, the arm endpoint position $y$ is given by the following transfer function of actuator torque $\tau$ and reaction force $F_d$:

$$y(s) = \frac{L\tau(s) + L^2 F_d(s)}{s(Js + b)}$$

Use the following transfer function for the rest of the questions:

$$y(s) = \frac{\tau + 2F_d}{s(s+10)}$$
(2-b). Consider a Proportional-plus-Derivative controller, as shown in Figure 3. We want to tune this control system so that the settling time will be $T_s = 400 \text{ ms}$ and the damping ratio will be $\zeta = 0.5$. Obtain the closed loop poles as well as the proportional gain $K$ and the derivative gain $k_D$ for the desired settling time and damping ratio.

![Figure 3 Proportional-plus-Derivative control of the endpoint y](image)

(2-c). For the PD control system designed in part (2-b), obtain the steady-state error for a unit step input as well as the one for a unit step disturbance $F_d$.

(2-d) Design a cascade dynamic compensator that meets the following specifications:
- The steady-state error for a step disturbance is zero.
- The steady-state error for a unit ramp disturbance is 5%.
- The transient response improved in part (2-b) will not significantly be changed.

Obtain the poles and zeros of the combined dynamic compensator including the above PD control and this new compensator. Draw the root locus of the total system, and discuss the result.