• Relax!

• This is a closed-book exam.

This problem deals with the dynamics and control of an Atomic Force Microscope (AFM). Figure 1 shows an illustration of such a device. A small micro-cantilever probe comes in contact with the sample surface and moves in the lateral direction (left-right). The bumps and valleys on the sample surface (topography) vary the deflection of the micro-cantilever probe. In atomic force microscopy however, it is desirable to maintain the cantilever deflection constant at a pre-set deflection value. The sample, to be scanned, is placed on top of a piezo actuator as shown in the Figure. The sample is moved in the vertical direction in order to continuously adjust the vertical position of the sample with respect to the probe and keep the deflection constant. The command signal to the piezo actuator is the voltage input, \( V_{in} \). The deflection of the micro-cantilever is measured by projecting a laser beam to the backside of the cantilever and observing the location of the reflection point on a photodiode, and measured as the voltage \( V_{out} \).

![Diagram of AFM setup]

Figure 1. A piezo actuator adjusts the probe/sample distance. \( V_{in} \) (voltage input to the piezo actuator) commands the piezo vertical expansion, and \( V_{out} \) (measured voltage output of the photo diode) reflects the deflection of the micro-cantilever.

Figure 2 illustrates the control loop for the displacement of the sample in the vertical direction. \( r \) is the pre-set deflection reference. \( e \) is the deflection error. \( G_c(s) \) represents the controller and \( G(s) \) the transfer function from \( V_{in} \) to \( V_{out} \) (see Figure 1). The disturbance, \( d \), represents the topography of the sample surface and disturbs the deflection of the cantilever as the probe is scanned in the lateral direction. \( n \) is the noise rooted in the electronics of the photo-diode.
Figure 2. Block diagram representation of the control loop for AFM operation. This control loop is responsible for the proper movement of the sample in the vertical direction to achieve a constant cantilever deflection.

**Question 1:** Figure 3 illustrates the frequency response of the piezo actuator with $V_{in}$ as the input and $V_{out}$ as the measured response. This plot can be achieved by driving the piezo actuator with a wideband excitation, $V_{in}$, and by simultaneously measuring the photodiode output, $V_{out}$, when the control loop of figure 2 is open, the probe is laterally stationary and directly in touch with the piezo actuator top (no sample). Ignore the dynamics of the photodiode, and the cantilever, i.e. assume that cantilever tip is always in touch with the surface and the photodiode voltage output is always proportional to cantilever deflection (displacement of the cantilever tip in the vertical direction). Provide a second order model that best reflects the system behavior up to the frequency bound, $\omega_b$, denoted by the vertical dotted line, with $V_{in}$ and $V_{out}$ as the input and output respectively. Note that the measured data includes time-delay (here a measurement artifact) which we do not want to include into the model. Find approximate numerical values for the parameters of this model. Round the values read from the plot if necessary to save time. Show how you arrive at your answer and explain the assumptions inherent in your approach. (4 points)
Figure 3. Measured frequency response of the piezo actuator with voltage excitation to the piezo, $V_{in}$, as input and cantilever deflection response measured on the voltage output of the photo diode, $V_{out}$, as output.

**Question 2:** The AFM control design consists of maintaining the output, $y$, constant as set by the reference value, $r$, in the presence of the disturbance, $d$, and noise, $n$. Briefly discuss how the following two different scenarios would affect your control design strategy: a) the frequency content of the noise and that of the disturbance overlap, and b) the frequency content of the noise does not overlap with that of the disturbance. (2 points)

**Question 3:** Assume that disturbance and noise fall on different frequency ranges. Disturbance, $d$, has an expected maximum bandwidth of $\omega_d$ with $\omega_d < 10^2$ Hz. Also assume that the noise $n$ falls over a much higher frequency range with respect to the disturbance i.e. $\omega_n > 10^4$ Hz, where $\omega_n$, is the minimum frequency associated with the noise. Does a closed loop transfer function given as, $T = \frac{1}{1 + \tau_c s}$ with, $\omega_d \ll \frac{1}{\tau_c} \ll \omega_n$, achieve the objective of maintaining the error small, $e \approx 0$, and simultaneously rejecting noise from the output? If yes show how, and if not explain why? (6 points)

**Question 4:** Assume you can either apply a PID control $G_c(s) = k_p \left( 1 + \frac{1}{\tau_1 s} + \tau_3 s \right)$ or a PI control, $G_c(s) = k_p \left( 1 + \frac{1}{\tau_1 s} \right)$, for the given system. Indicate which of the two controllers can achieve the closed loop transfer function, $T$, given above. Find the corresponding controller parameters in terms of $\tau_c$ and the parameters of the model found in Question 1 such that it leads to the given closed loop transfer function, $T = \frac{1}{1 + \tau_c s}$. (4 points)
**Question 5**: For faster imaging we need to move the micro-cantilever in the lateral (left-right) direction faster. This would in return extend the frequency range associated with the disturbance i.e. increase $\omega_d$. Using a PID controller for the above problem is there a limit on how fast one can take AFM images before the system goes unstable? Regardless of your answer (either Yes or No) explain in detail whether it is logical from a practical point of view. What are the factors that in practice may limit the maximum possible imaging speed? (4 Points)

**Total**: 20 points