This is a closed-book exam

Consider the magnetically suspended ball shown in the above figure. The forces acting on the ball are (a) the magnetic force \( \alpha \frac{\partial^2 y}{\partial t^2} \), in which \( \alpha \) is related to the number of windings, \( i \) is the current in the windings, and \( y \) is the distance of the ball from the magnet; and (b) the force of gravity \( mg \), in which \( m \) is the mass of the ball. Assume that the sensor can exactly measure the position of the ball \( y(t) \) and that the control input is the current \( i(t) \). The control objective is to stabilize the position of the ball \( y(t) \) at a value about a nominal constant value \( y_N \). To this end, answer the following questions
(1) (System model–3 points) Write the dynamical model that represents the behavior of the system (1 point). In order to tackle the control objective linearize the system about the steady state given by \( y = Y_N \) (and the corresponding nominal input \( I_N \)) and write the plant transfer function (2 points).

(2) (Stabilization–3 points) Assuming that you found in (1) that the linearized plant transfer function is of the form \( P(s) = \frac{k}{s^2 + ps} \), design a closed loop feedback control \( C(s) \) to stabilize the system and determine the steady state error to a step reference input (3 points).

(3) (Steady state error– 3 points) If the steady state error is not zero, modify your compensator so that (a) the steady state error is zero and (b) the overshoot to a step input is minimized (3 points).

(4) The original system is nonlinear. Highlight the main limitations of your linear feedback control design (1 point).