An elastic shaft carries a centered circular disk of radius \( r \) bonded to the shaft. The disk mass \( m \) is large in comparison with the mass of the shaft. The shaft is supported by high quality ball bearings. When the disk is given a small transverse displacement and released it vibrates transversely with a natural frequency \( \omega_m \).

(i) A fixed circular structure of radius \( R \) is erected concentrically around the disk. Estimate the magnitude of the transverse force \( F \) which would be required to push the disk into contact with the structure.

(ii) Now consider the shaft and disk to be rotating at the fixed rate \( \omega \) (an external synchronous motor maintains the speed). The clearance \( \varepsilon = R - r \) is sufficiently large that, under normal disturbances, vibrations, etc., the disk does not contact the surrounding structure. Your job here, however, is to consider what will happen in the case of a disturbance large enough to cause contact. Suppose moreover that friction is sufficient to maintain pure rolling contact (no-slip). Which way does the disk roll and how fast does it go? What is the rate \( \Omega \) at which the disk center whirls about the line BB joining the bearings?

(iii) In order for this whirlng to be maintained the centrifugal force acting on the disk must be equal to or greater than the force \( F \) calculated in (i) above. What is the minimum speed \( \omega \) for which this is assured?